Analytical and Numerical Modeling of Production From Unconventional Resource Plays

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Economic success of unconventional resource plays and **advanced multi-staged hydraulic fracture horizontal well technology.**
The Key Characteristics of Production from Unconventional Reservoirs:

- Ultra-tight, low permeable sediment rocks (< 1mD)
- Nanoscale pore sizes
- Naturally fractured formation
- Hydraulic fracturing stimulation
- Complex fracture networks (natural, induced, hydraulic fractures)
PART I:
Multi-Physics Numerical Simulation of Shale Gas Plays
Key Simulation Features

Physics

- Two phase flow (Gas/Water)
- Non-Darcy flow of gas
- Gas adsorption/desorption
- Klinkenberg effect
- Capillary pressure

Gridding and Discretization

- Discrete Fracture Modeling of hydraulic fractures
- Centroidal Voronoi grid
- Fully Implicit (ADETL)
- TPFA / MPFA frameworks
Delaunay Triangulation

- Analogy between mesh and truss structure
- Delaunay Triangulation based on equilibrium of forces

\[
f(l, l_0) = \begin{cases} 
  k(l_0 - l) & \text{if } l < l_0 \\ 
  0 & \text{if } l \geq l_0 
\end{cases}
\]

To solve \( \vec{F}(\vec{p}) = 0 \)

\[
\begin{align*}
\frac{dp}{dt} &= F(p) \\
p_{n+1} &= p_n + \Delta tF(p_n)
\end{align*}
\]
Centroidal Voronoi Tessellation

1. Initial set of points
2. Construct Voronoi tessellation
3. Compute mass centroids
4. Move points
5. Check criterion
   - Y: End
   - N: Repeat steps 2-4
Explicit Representation of Hydraulic Fractures
Multi-Physics Flow and Transport

- Non-Darcy flow

\[-(\nabla \Phi_\beta) = \frac{\mu_\beta}{k_{r\beta}} k v_\beta + \beta \rho_\beta v_\beta \mid v_\beta \mid \quad \Rightarrow \quad v_\beta = -k \delta_\beta \frac{k_{r\beta}}{\mu_\beta} \left( \nabla \Phi_\beta \right) \]

\[\delta_\beta = \left( I + k \beta \rho_\beta \frac{k_{r\beta}}{\mu_\beta} \mid v_\beta \mid \right)^{-1}\]

- Gas adsorption/desorption

\[m_g = \rho_R \rho_g V_E \quad V_E = V_L \frac{p}{p + P_L}\]

- Klinkenberg effect

\[k_g = k \left( 1 + \frac{b}{p_g} \right)\]
Fully Implicit Discretization

\[
\left\{ (\phi \rho S)_{i}^{\beta,n+1} + m_{i}^{\beta,n+1} - (\phi \rho S)_{i}^{\beta,n} - m_{i}^{\beta,n} \right\} \frac{V_{i}}{\Delta t} = \sum_{j \in \eta_{i}} \text{flow}_{ij}^{\beta,n+1} + Q_{i}^{\beta,n+1}
\]

Residual:

\[
R_{i}^{\beta,n+1} = \left\{ (\phi \rho S)_{i}^{\beta,n+1} + m_{i}^{\beta,n+1} - (\phi \rho S)_{i}^{\beta,n} - m_{i}^{\beta,n} \right\} \frac{V_{i}}{\Delta t} - \sum_{j \in \eta_{i}} \text{flow}_{ij}^{\beta,n+1} + Q_{i}^{\beta,n+1} = 0
\]

\(\beta\): gas or water phase

\(n\): time level

- Highly nonlinear system of equations
- Computation of Jacobian using ADETL
Ongoing Work

- Hybrid simulation models: Discrete Fracture Model + Dual Permeability
- Multi-Point Flux Approximation discretization
PART II:
Pressure Transient Analysis of Multi-Stage Hydraulic Fracture Tight Oil Reservoirs
Complex fracture networks of micro-seismic events

Physical fracture network system

sugar cube model of natural fractures network
Quad-linear Flow Model

- **Region I:** Hydraulic Fracture Region
- **Region II:** Effective Fracture Network Area (EFNA)
- **Region III:** Outer Reservoir Region
- **Region IV:** Unstimulated Reservoir Region
Mathematical Model, Region I

Fracture Linear Flow:

\[
\frac{\partial^2 p_{1D}}{\partial x_D^2} + a_1 \frac{\partial p_{2D}}{\partial y_D}\bigg|_{y_D=w_D} = b_1 \frac{\partial p_{1D}}{\partial t_D}
\]

where,

\[
a_1 = \frac{x_f k_f}{w_f k_1}
\]

and,

\[
b_1 = \frac{\phi c_i k_f}{(\phi c_i)_2 k_1}
\]

Boundary Conditions:

\[
\left. \frac{\partial p_{1D}}{\partial x_D} \right|_{x_D=0} = 0
\]

\[
\left. \frac{\partial p_{1D}}{\partial x_D} \right|_{x_D=0} = c_1 \left( 1 - C_{Df} \frac{\partial p_{wD}}{\partial t_D} \right)
\]

\[
P_{wD} = p_{1D} \big|_{x_D=0} - S \left. \frac{\partial p_{1D}}{\partial x_D} \right|_{x_D=0}
\]
Mathematical Model, Region II

Dual-Porosity Formulation in the Stimulated Reservoir Region:

\[
\frac{\partial^2 p_{fD}}{\partial y_D^2} + \lambda \left( p_{mD} - p_{fD} \right) + \frac{k_m}{k_f} \frac{\partial p_{3D}}{\partial x_D} \bigg|_{x_D=1} + \frac{k_m}{k_f} \frac{x_f}{l} \frac{\partial p_{3D}}{\partial y_D} \bigg|_{y_D=l_D} = \omega \frac{\partial p_{fD}}{\partial t_D}
\]

where

\[
\lambda = \alpha \frac{k_m}{k_f} x_f^2 \quad \text{and,} \quad \omega = \frac{C_{if} \phi_f}{C_{im} \phi_m + C_{if} \phi_f}
\]

Boundary Conditions:

\[
P_{2D} \bigg|_{y_D=w_D} = P_{1D} \bigg|_{y_D=w_D}
\]

\[
\frac{\partial p_{2D}}{\partial y_D} \bigg|_{y_D=l_D} = \frac{k_m}{k_f} \frac{\partial p_{4D}}{\partial y_D} \bigg|_{y_D=l_D}
\]
Mathematical Model, Region III

Linear Flow in the Outer Reservoir Region:

\[
\frac{\partial^2 p_{3D}}{\partial x_D^2} = \frac{k_f \phi_m c_{tm}}{k_m (\phi c_i)^2} \frac{\partial p_{3D}}{\partial t_D}
\]

Boundary Conditions:

\[
\frac{\partial p_{3D}}{\partial x_D} \bigg|_{x_D=x_D} = 0
\]

\[
p_{2D} \bigg|_{x_D=1} = p_{3D} \bigg|_{x_D=1}
\]
Mathematical Model, Region IV

Linear Flow in the Unstimulated Reservoir Region:

\[
\frac{\partial^2 p_{4D}}{\partial y_D^2} = \frac{k_f \phi_m c_m}{k_m (\phi c_i)^2} \frac{\partial p_{4D}}{\partial t_D}
\]

Boundary Conditions:

\[
p_{2D} \bigg|_{y_D=l_D} = p_{4D} \bigg|_{y_D=l_D}
\]

\[
\frac{\partial p_{4D}}{\partial y_D} \bigg|_{y_D=y_{eD}} = 0
\]
Laplace Solution

Transforming the equations into Laplace domain and solving them simultaneously for well pressure, yields:

\[
P_{wD} = \frac{c_1}{s \sqrt{F_4 \tanh(\sqrt{F_4})}} + \frac{s c_1}{s} \\
= \frac{c_1}{s \sqrt{F_4 \tanh(\sqrt{F_4})}} + \frac{s c_1}{s} \left( \frac{c_1}{s \sqrt{F_4 \tanh(\sqrt{F_4})}} + \frac{s c_1}{s} \right) s^2 C_{Df} - 1
\]

The well pressure solution in time domain is obtained from numerical inversion of the solution in Laplace domain using Stehfest numerical inversion algorithm.
Model Validation I

When $l_D \rightarrow 1$, the quad-linear flow model results match those of the tri-linear flow model proposed by Brown, Ozkan, Raghavan, and Kazemi, 2009.
Model Validation II

Comparison with commercial simulator (Eclipse)

- Grid dimension, 200x200x2
- $k_m$, 0.23 mD
- $k_f$, 200 mD
- Hydraulic fracture permeability, 2000 mD
Model Validation II

Comparison with commercial simulator (Eclipse)
Model Validation II

Comparison with commercial simulator (Eclipse)

![Graph comparing simulation and analytical solutions](image)
Sensitivity Studies

Size of the Stimulated Reservoir Region, $l$

- Matrix permeability, $k_m$ (mD): 0.23
- Micro-fracture permeability, $k_f$ (mD): 200
- Hydraulic fracture length, $x_f$ (m): 200
- Hydraulic fracture permeability, $k_1$ (mD): 2000

Dimensionless Pressure & Derivative $p_D', p_D$

Dimensionless Time $t_D$
Ongoing Work

- Application of the quad-linear flow model for tight gas reservoirs by the use of pseudo-pressure
- Application of Fractal Diffusivity models combined with the quad-linear flow model
Summary

- Ongoing work on development of a two phase flow simulator for shale gas reservoirs with a wide range of physical phenomena and explicit representation of hydraulic fractures.

- Using a quad-linear flow model, we were able to account for the Effective Fracture Network Area surrounding the hydraulic fractures.

- In the limit when the stimulated reservoir region covers the entire area between hydraulic fractures, the quad-linear flow model mimics the classic tri-linear flow model.

- Comparison of the analytical solution with numerical solution obtained from commercial simulator validates the model assumptions for a reasonable period of time.
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Dimensionless Variables

Pressure:

\[ p_{nD} = \frac{k_m h (p_i - p_n)}{q \mu} \quad n = 1, 2, 3, 4 \]

Time:

\[ t_D = \frac{k_f t}{(\phi c_t)_2 \mu x_f^2} \]

Spatial Parameters:

\[ x_D = \frac{x}{x_f} \quad y_D = \frac{y}{x_f} \]
\[ l_D = \frac{l}{x_f} \quad w_D = \frac{w}{x_f} \]
Effective Fracture Network Area (EFNA)

micro-seismic fracture mapping data of well A27, first stage fracture treatment, Changqing tight oil field, China.

FCI Chart with different fractured horizontal wells, Changqing tight oil field, China.

Local appearance of the Effective Fracture Network Area (EFNA) with high permeability around hydraulic fracture stages.
Sensitivity Studies

Interporosity Flow Coefficient, $\lambda$

![Graph showing the relationship between dimensionless pressure and derivative and dimensionless time. The graph plots different scenarios with values of $\lambda$.](image)

- $\lambda = 0.1$
- $\lambda = 0.2$
- $\lambda = 0.4$

Dimensionless Pressure & Derivative $P'_D, P_D$

Dimensionless Time $t_D$

$10^0$ $10^1$ $10^2$ $10^3$ $10^4$ $10^5$

$10^0$ $10^1$ $10^2$ $10^3$ $10^4$ $10^5$
Sensitivity Studies

Storativity Ratio, $\omega$

![Graph showing storativity ratio for different values of $\omega$.]